

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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Pearson Edexcel International GCSE

Time 2 hours

Paper
reference

4PM1/02R

Further Pure Mathematics

PAPER 2R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ►

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Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1 - r^n)}{(1 - r)}$

Sum to infinity, $S_\infty = \frac{a}{1 - r} \quad |r| < 1$

Binomial series

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad \text{for } |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC : $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Question 2 continued

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(Total for Question 2 is 8 marks)



Question 3 continued

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(Total for Question 3 is 6 marks)



Question 4 continued

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Question 4 continued

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Question 4 continued

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(Total for Question 4 is 10 marks)



5 A geometric series G has first term 12 and common ratio $\frac{3}{8}$

(a) Find the sum to infinity of G (2)

(b) Show that the 6th term of G can be written as $\frac{3^6}{2^{13}}$ (3)

The n th term of G is u_n

(c) By finding an expression for u_n in terms of n , show that

$$\log_2 u_n = n \log_2 3 - 3n + 5 \quad (5)$$



Question 5 continued

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Question 5 continued

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(Total for Question 5 is 10 marks)



6 The curve C has equation

$$y = 4\sqrt{x}$$

The point A on C has coordinates $(9, 12)$

The tangent to C at the point A meets the x -axis at the point T

(a) Find the coordinates of T (5)

The normal to C at the point A meets the x -axis at the point N

(b) Find the coordinates of N (4)

(c) Calculate the area of triangle ATN (2)

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Question 6 continued

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(Total for Question 6 is 11 marks)



7 Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

(a) show that $\cos \theta = -\frac{1}{2}$

(4)

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x \quad \text{for } 0^\circ \leq x < 180^\circ$$

(4)

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Question 7 continued

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(Total for Question 7 is 8 marks)



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8 Liquid drips onto a large horizontal flat cloth, forming a circular stain.

The liquid starts to drip onto the cloth at time $t = 0$

The area of the stain increases at a constant rate of $1.5 \text{ cm}^2/\text{s}$

(a) Find, in terms of π , the radius of the stain at time $t = 4$ seconds. (3)

(b) Find, in cm/s to 3 significant figures, the rate at which the radius of the stain is increasing at time $t = 4$ seconds. (4)

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Question 8 continued

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(Total for Question 8 is 7 marks)



- 9 Given that α is the acute angle such that $\tan \alpha = \frac{2}{3}$

(a) find the exact value of $\cos \alpha$

(1)

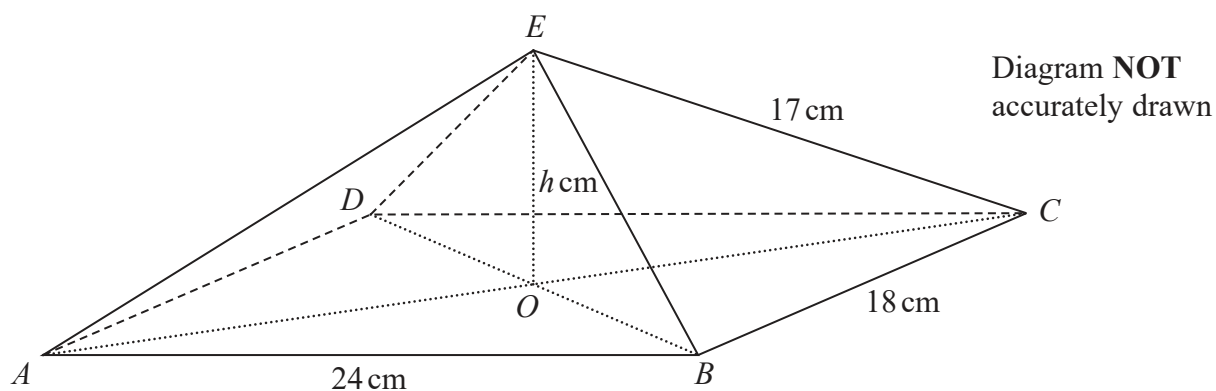


Figure 1

Figure 1 shows a right pyramid with a rectangular base $ABCD$ and vertex E

The rectangular base of the pyramid is horizontal with $AB = 24$ cm and $BC = 18$ cm.

The diagonals of the base intersect at the point O

The vertex E of the pyramid is vertically above O such that

$$AE = BE = CE = DE = 17 \text{ cm}$$

The height of the pyramid is h cm.

(b) Find the value of h

(3)

The size of the angle between the plane EBC and the plane $ABCD$ is θ°

(c) Show that $\tan \theta^\circ = \frac{2}{3}$

(2)

The point P is the midpoint of EB and the point Q is the midpoint of EC

(d) Find the size, in degrees to one decimal place, of the angle between the plane OPQ and the plane $BCQP$

(4)



Question 9 continued

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



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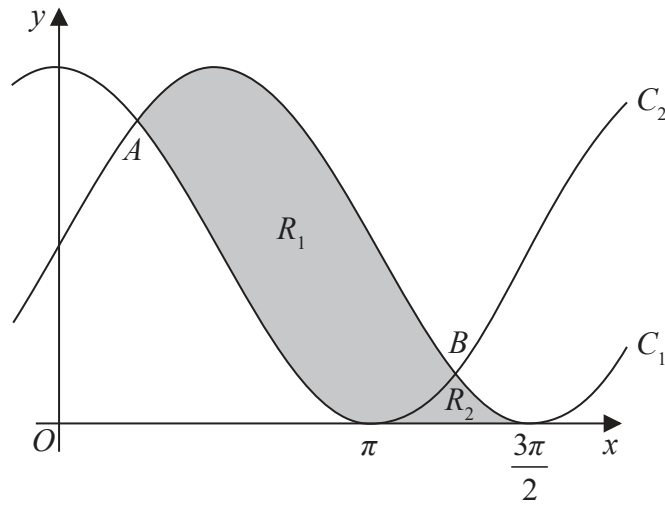


Diagram NOT accurately drawn

Figure 2

Figure 2 shows part of the curve C_1 with equation $y = \sin x + 1$ and part of the curve C_2 with equation $y = \cos x + 1$

As shown in Figure 2, C_1 and C_2 intersect at the point A and at the point B

- (a) Find the exact value of the x coordinate of A and the exact value of the x coordinate of B (3)

The shaded finite region R_1 shown in Figure 2 is bounded by C_1 and C_2

The shaded finite region R_2 shown in Figure 2 is bounded by the x -axis, C_1 and C_2

- (b) Use calculus to find the ratio

$$\text{area of } R_1 : \text{area of } R_2$$

Give your answer in the form $a : \left(\frac{\pi\sqrt{2}}{b} - c \right)$ where a , b and c are integers.

(9)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 12 marks)



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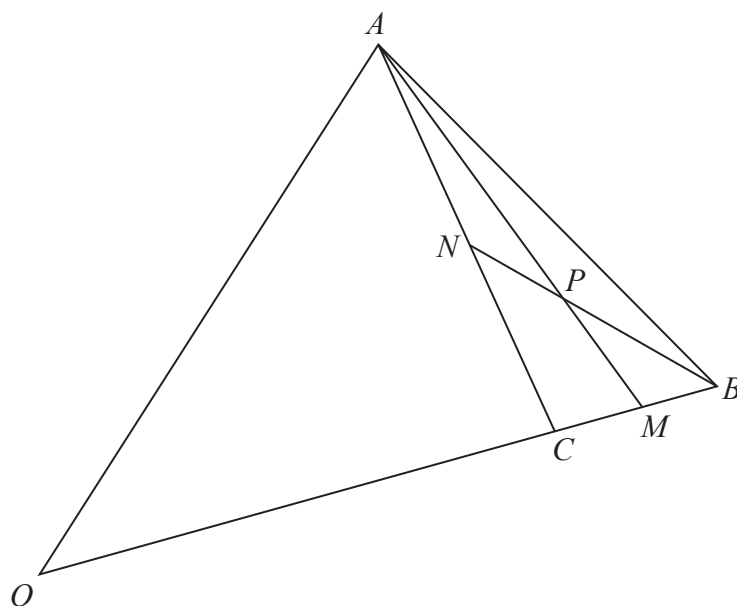


Diagram NOT
accurately drawn

Figure 3

Figure 3 shows triangle OAB with $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$

The point C lies on OB such that $OC:CB = 2:1$

The point M is the midpoint of CB and the point N is the midpoint of AC

The lines AM and NB intersect at the point P

- (a) Using a vector method, find \vec{OP} as a simplified expression in terms of \mathbf{a} and \mathbf{b} (9)

The point Q is the midpoint of AB

- (b) Using a vector method, show that C , P and Q are collinear. (4)



Question 11 continued

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Question 11 continued

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(Total for Question 11 is 13 marks)

TOTAL FOR PAPER IS 100 MARKS

